ATOC 5051 INTRODUCTION TO PHYSICAL OCEANOGRAPHY

Lecture 14

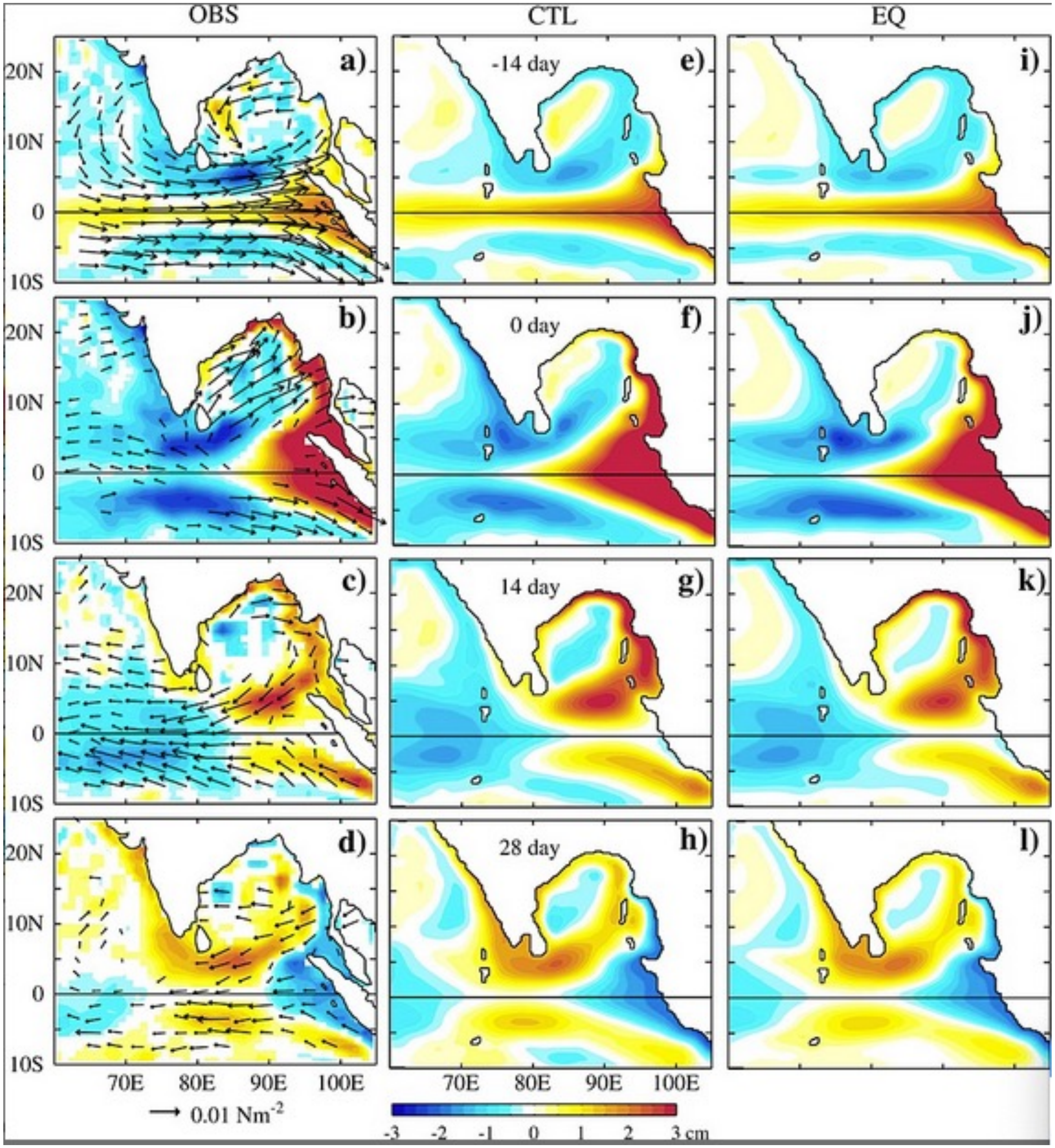
Learning objectives:

understand the characters (e.g., energy dispersion, solution structure) of the following waves:

1. Coastally trapped waves
2. Equatorial waves:

(i) Equatorial Kelvin wave

1. Effects of side boundaries: coastally-trapped waves



Lag regression (Lags indicated on the middle column) of 20–150 day filtered QuikSCAT wind stress (first column) and sea level to the normalized principal component of the first EOF of 20–150 day filtered sea level in (a–d) observations (17% of total variance, second mode 6%), (e–h) CTL experiment (44% of total variance, second mode 14%), and (i–l) EQ experiment (49% of total variance, second mode 15%). The regression at lag 0 in Figures 1b, 1f, and 1j shows the spatial structure of the first EOF.

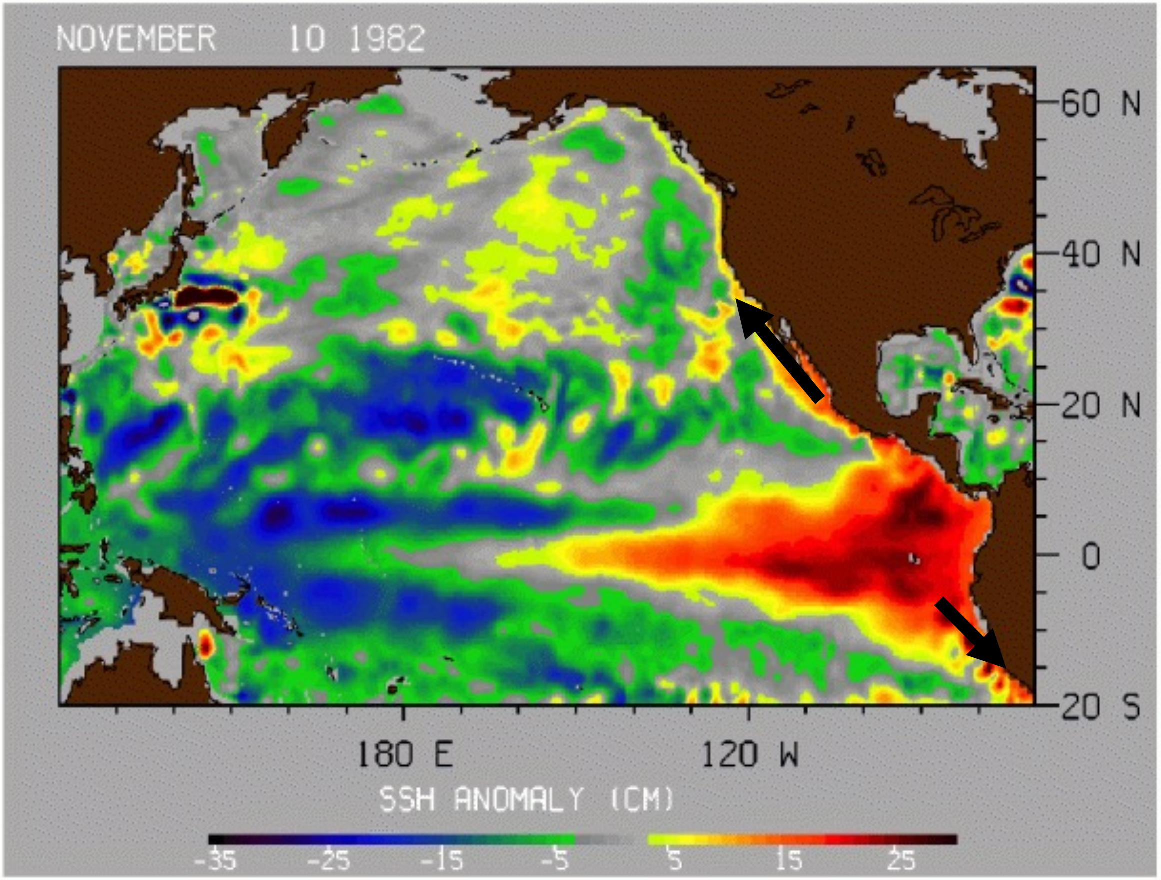
Suresh I., J. Vialard, M. Lengaigne W. Han, J. McCreary, F. Durand, P.M. Muraleedharan, 2013: Origins of wind-driven intraseasonal sealevel variations in the North Indian Ocean coastal waveguide. Geophys. Res. Lett., 40, 5740-5744,

Effects of side boundaries: coastally-trapped waves

Previous classes:
gravity waves &
midlatitude Rossby waves
(surface & internal) **- open ocean.**

Today: effects of side boundaries:
Follow the same procedure as done for open ocean waves: dispersion relation & wave solution structure

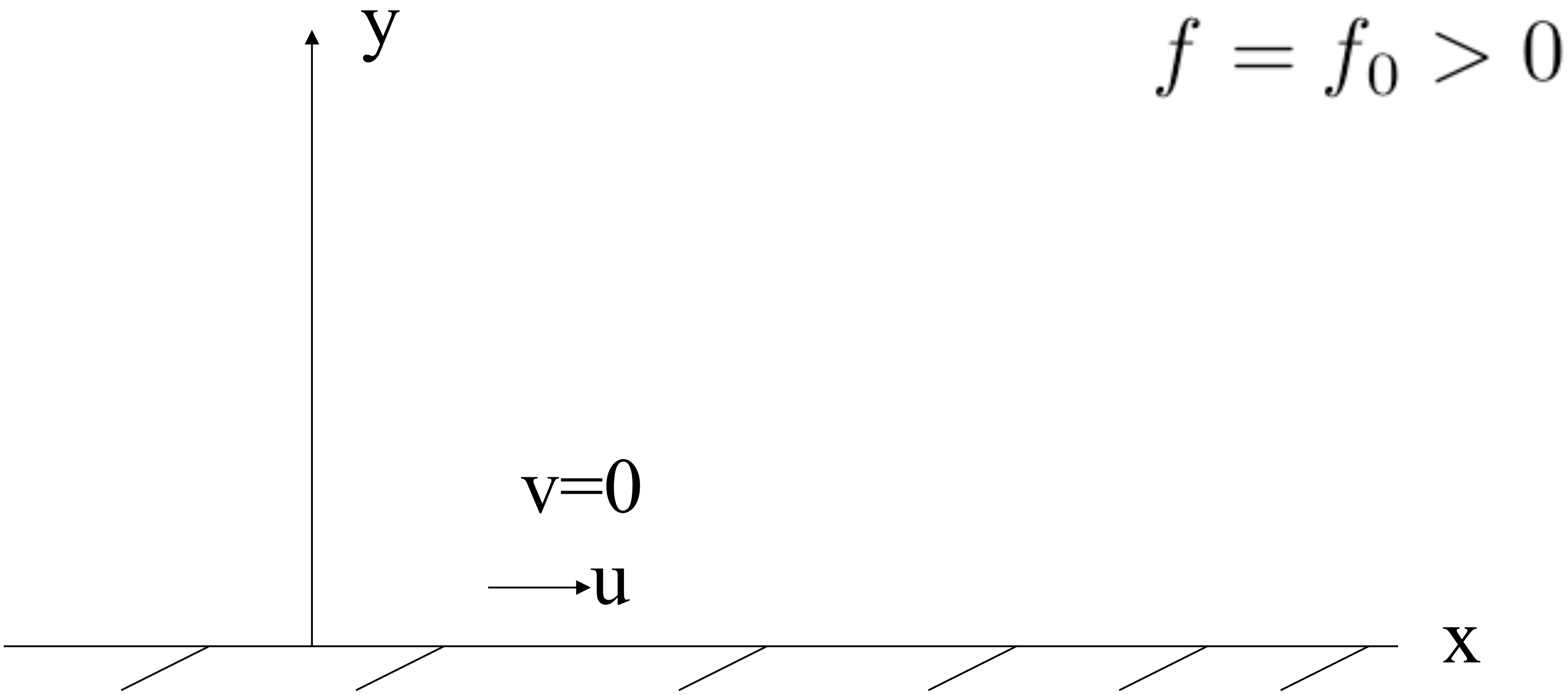
1982 El Nino



Coasts act as waveguides.

El Niño Gives Glimpse Of Sea Level Rise On San Diego Beaches:
https://www.youtube.com/watch?v=--fC4tlTEQI

a) Coastal kelvin wave



$f = f_0 > 0$

Coast. Vertical walls.

we again use the shallow water equation with **constant f**: $f = f_0 > 0$

(Northern Hemisphere)

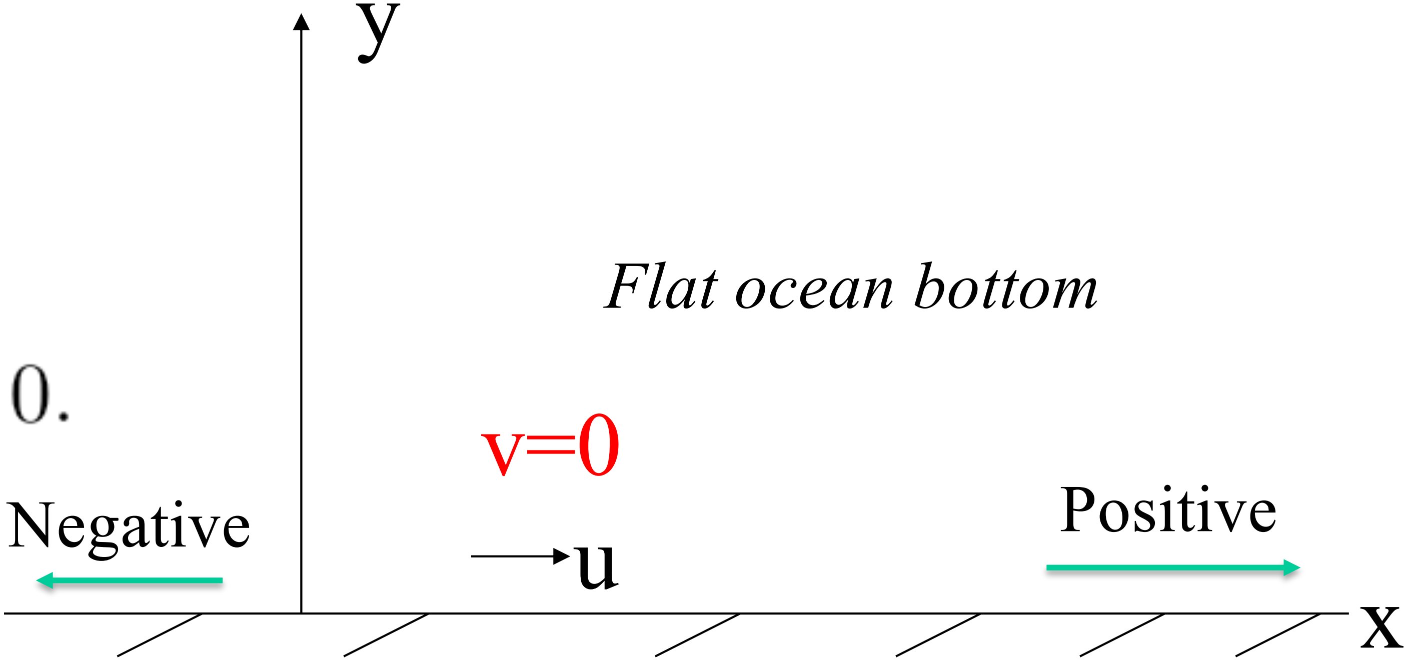
Find solutions for v=0, subject to boundary condition: $v = 0, @y = 0.$

v=0

$$\frac{\partial u}{\partial t} = -g\frac{\partial \eta}{\partial x},$$

$$fu = -g\frac{\partial \eta}{\partial y},$$

$$\frac{\partial \eta}{\partial t} + H\frac{\partial u}{\partial x} = 0.$$



y

Flat ocean bottom

v=0

Negative

u

Positive

x

From the above equations, we can obtain,

$$u_{tt} = gHu_{xx}$$

Assume wave form,

$$u = u(y)e^{i(kx-\omega t)}$$

We obtain

$$\omega = \pm kc,$$

where $c = \sqrt{gH}$

This is just like the dispersion relation for long, surface gravity waves in non-rotating system.

The existence of f does not affect the dispersion relation! What’s the effect of *f, then?*

$$\eta = \eta(y)e^{i(kx-\omega t)}, u = u(y)e^{i(kx-\omega t)},$$

The set of equations yields,

$$\eta_y = -\frac{fk}{\omega}\eta,$$ and choose $\omega = -kc,$

$$\longrightarrow \quad \eta_y = +\frac{f}{c}\eta,$$

Thus, $\eta = \eta_0 e^{+\frac{f}{c}y} e^{i(kx-\omega t)}.$

Since sea level increases with the increase of y, which is farther and farther away from the coast, it is not a reasonable solution because energy should decay away from the energy source; for this case the source is the coast.

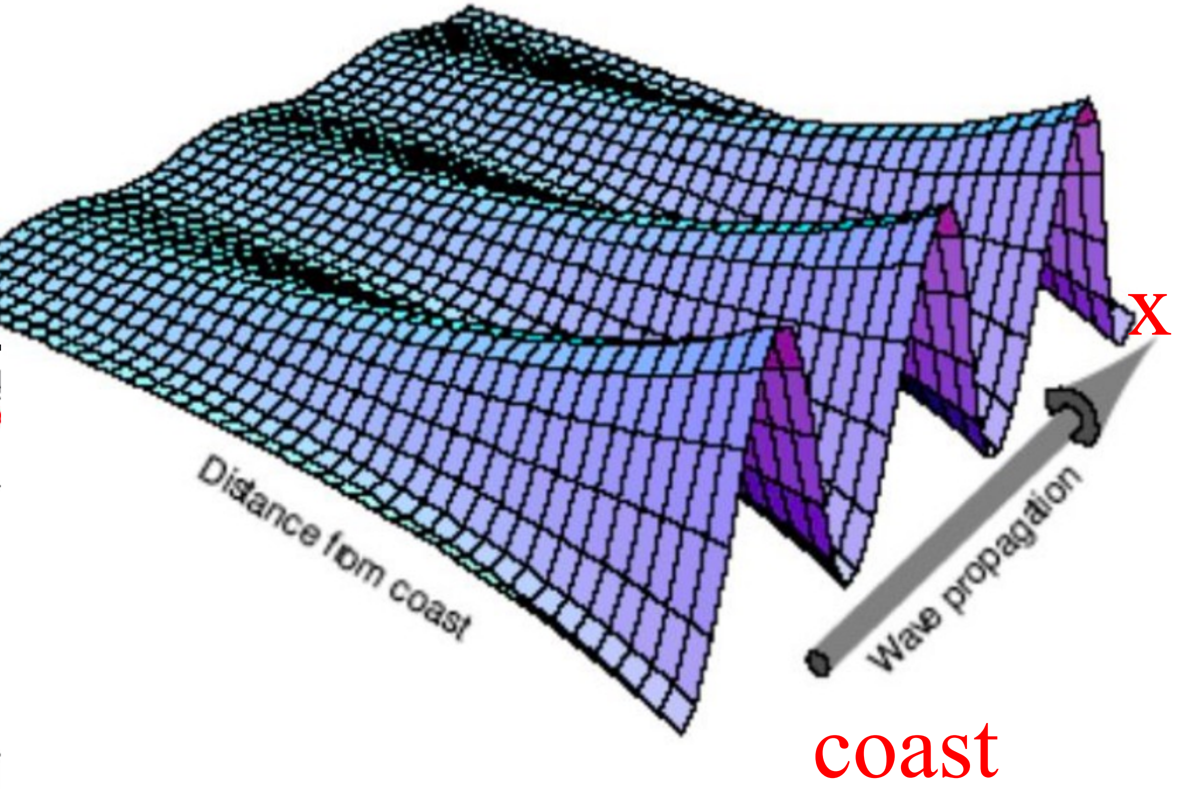
Choosing $\omega = kc,$ we obtain:

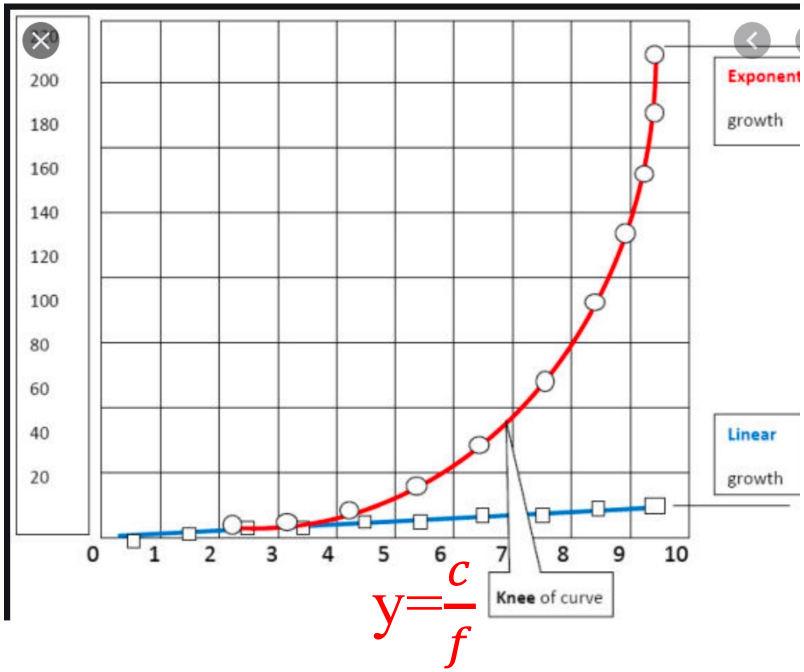
$$\eta = \eta_0 e^{-\frac{f}{c}y} e^{i(kx-\omega t)},$$

This solution obtains a maximum at the coast and decays away from it. Reasonable.

When y = $\frac{c}{f}, \eta = \eta_0 e^{-1} = \frac{\eta_0}{e}$

$$\approx \eta_0 37.8\%$$



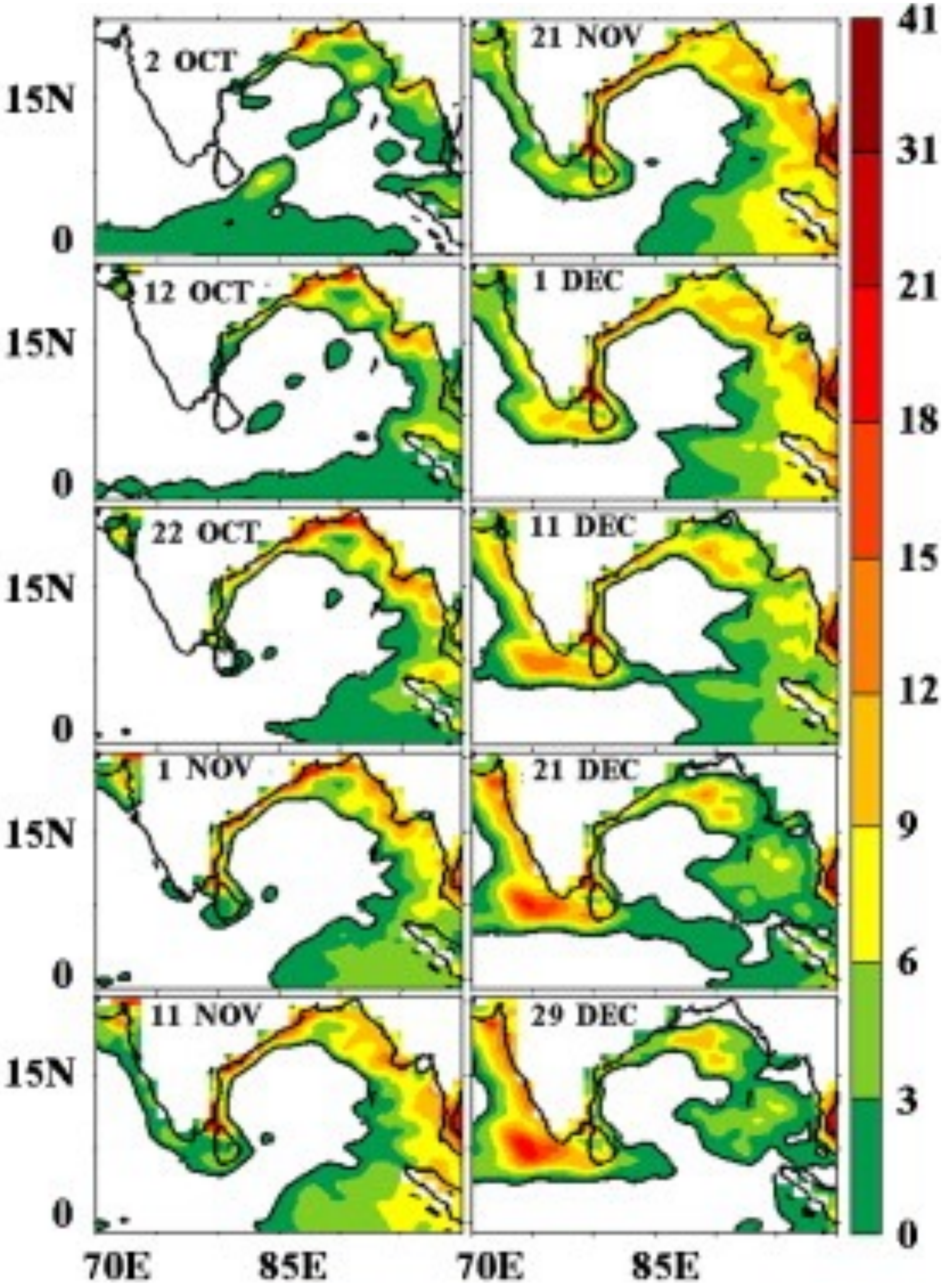


Key features:

Coastal Kelvin waves: propagate with the coast to its right (left) in Northern (Southern) Hemisphere.

Solutions are trapped to the coast, decaying away from it exponentially, with an e-folding scale of $\frac{c}{f}$, ***the Rossby radius of deformation.***

Satellite Observed Sea Surface Height anomalies (SSHA): Coastally-trapped waves in the Bay of Bengal of the Indian Ocean: (Multi-year mean) (*Rao et al. 2010, Deep Sea Research*)



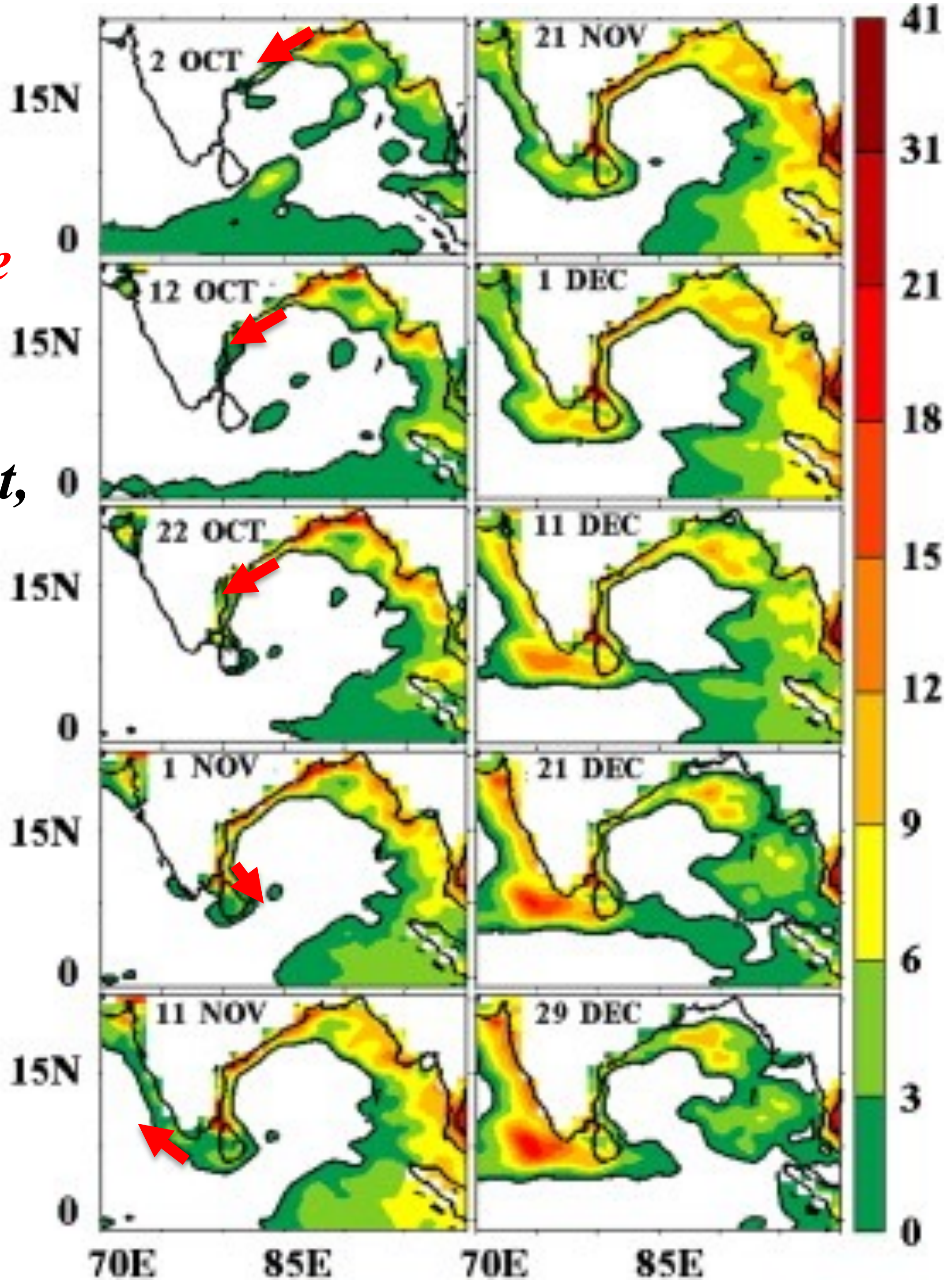
Group discussion:

- Identify coastal kelvin waves signals;
- Justify why you think they are coastal Kelvin waves

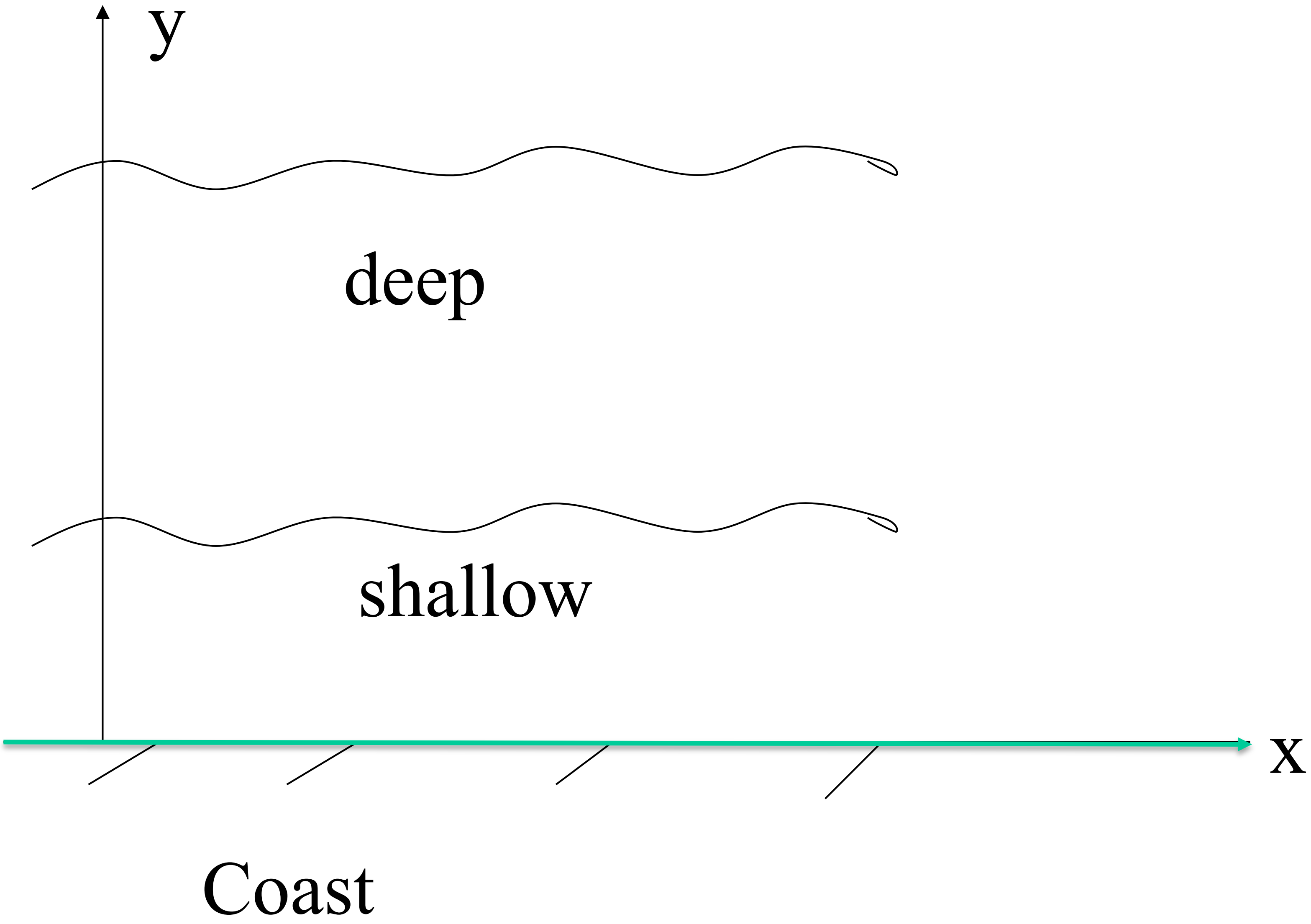
$$\eta = \eta_0 e^{-\frac{f}{c}y} e^{i(kx-\omega t)},$$

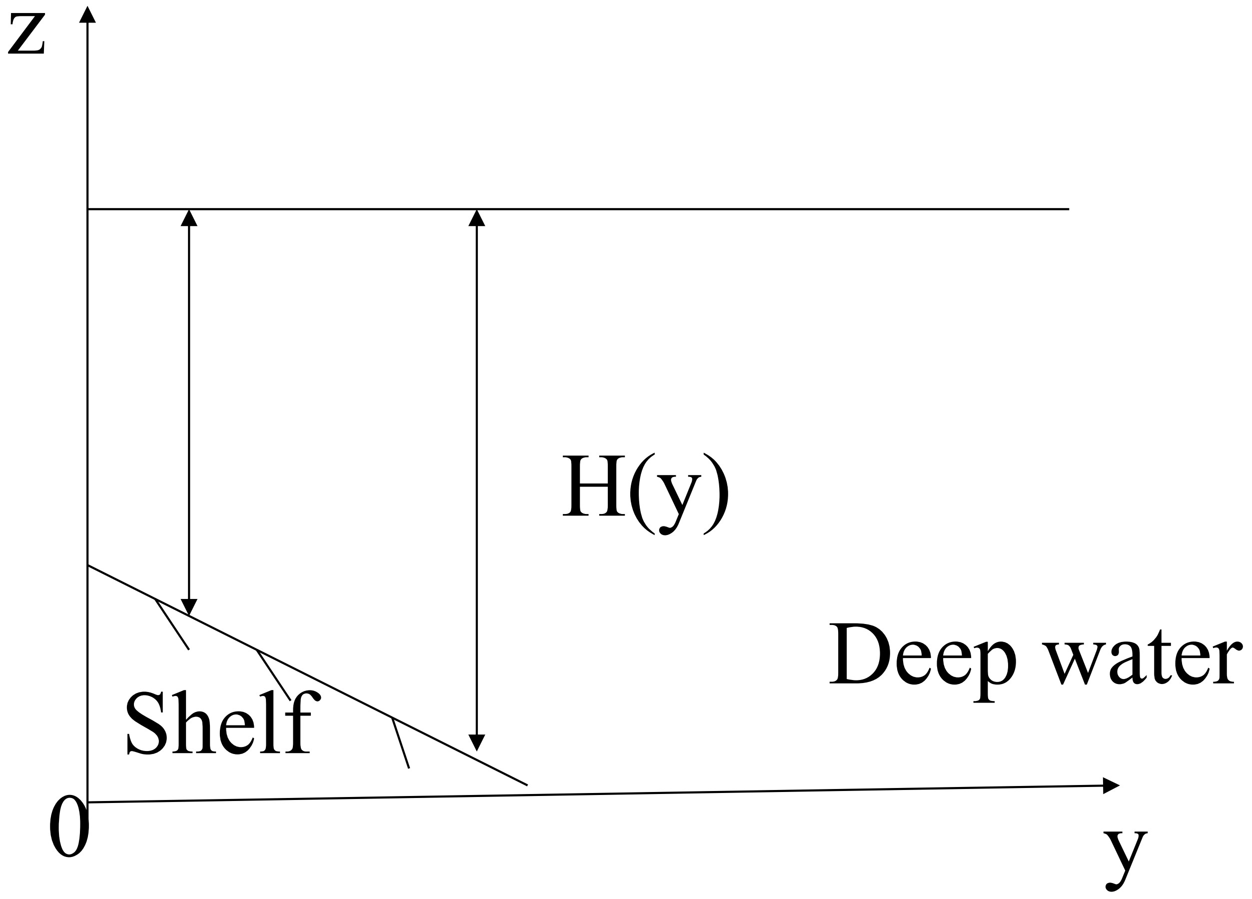
Coastal Kelvin waves: ***propagate with the coast to its right in Northern hemisphere.***

Solutions are trapped to the coast, decaying away from it exponentially, with an e-folding scale of $\frac{c}{f}$, ***the Rossby radius of deformation***.



Continental shelf waves: topographic Rossby waves




z
H(y)
Deep water
Shelf
0
y

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x},$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y},$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial (Hu)}{\partial x} + \frac{\partial (Hv)}{\partial y} = 0.$$

H: depth of water column. Analytic solution, more complicated.

$\frac{\partial H}{\partial y}$ acts as $\beta = \frac{\partial f}{\partial y}$

Referred to as topographic Rossby waves

Propagation direction: like coastal Kelvin waves

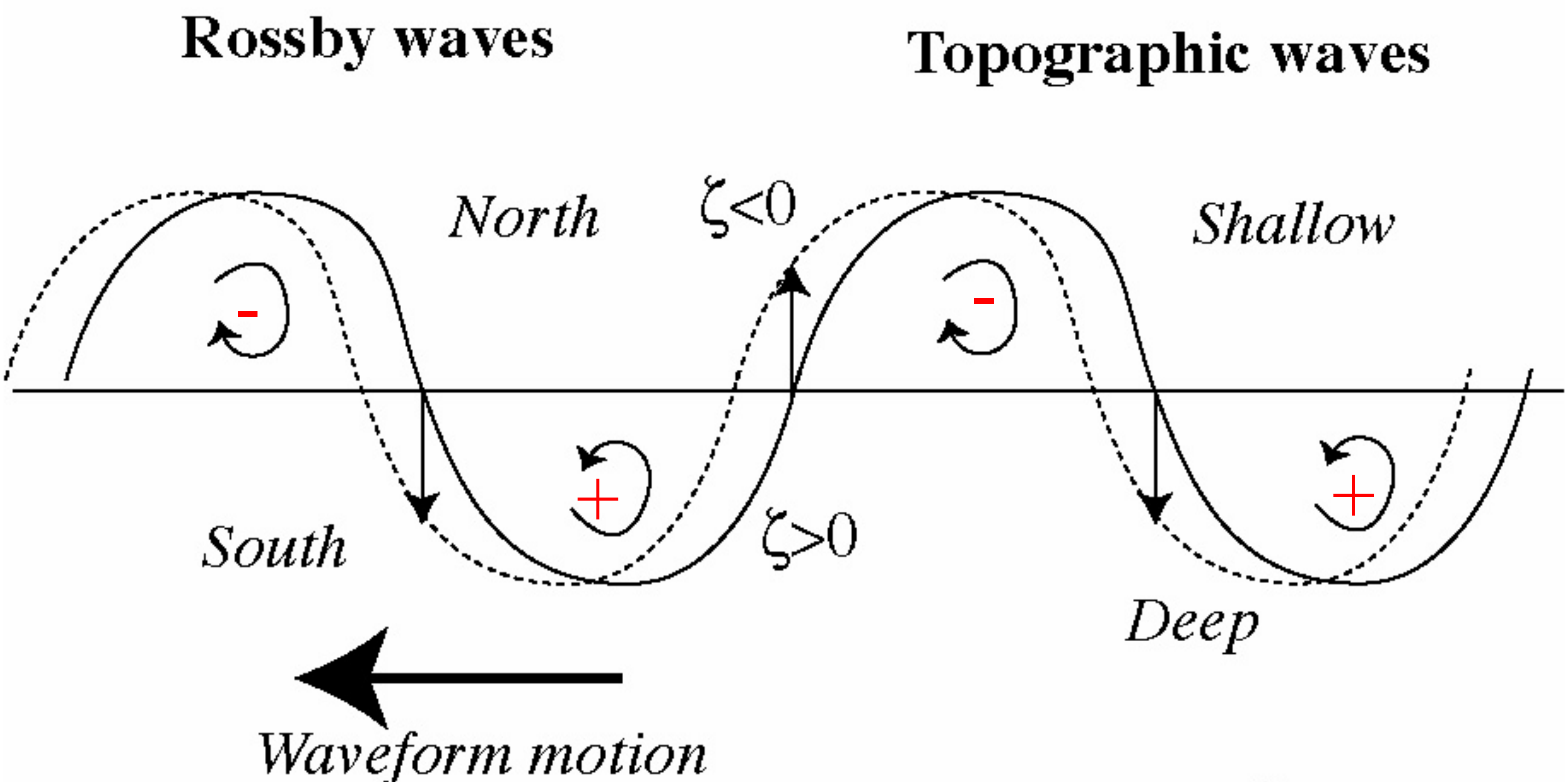
Shelf waves: dispersive.

Mechanism: like Rossby waves: Potential vorticity conservation:

$$PV = \frac{\zeta + f}{H} = constant$$

Here, H varies with y because of the shelf;
Near the coast, scale is small, $f \approx constant$

NH, North Coast



$$D/Dt\,(f+\zeta/h)=0$$

ation (what causes the Gulf Stream?)

2. Equatorial waves

(i) The equatorial Kevin wave

a) Dispersion relation:

$$\omega = kc,$$

barotropic or baroclinic mode speed;

Barotropic mode:

$$c = c_0 = \sqrt{gH} = 200m/s$$

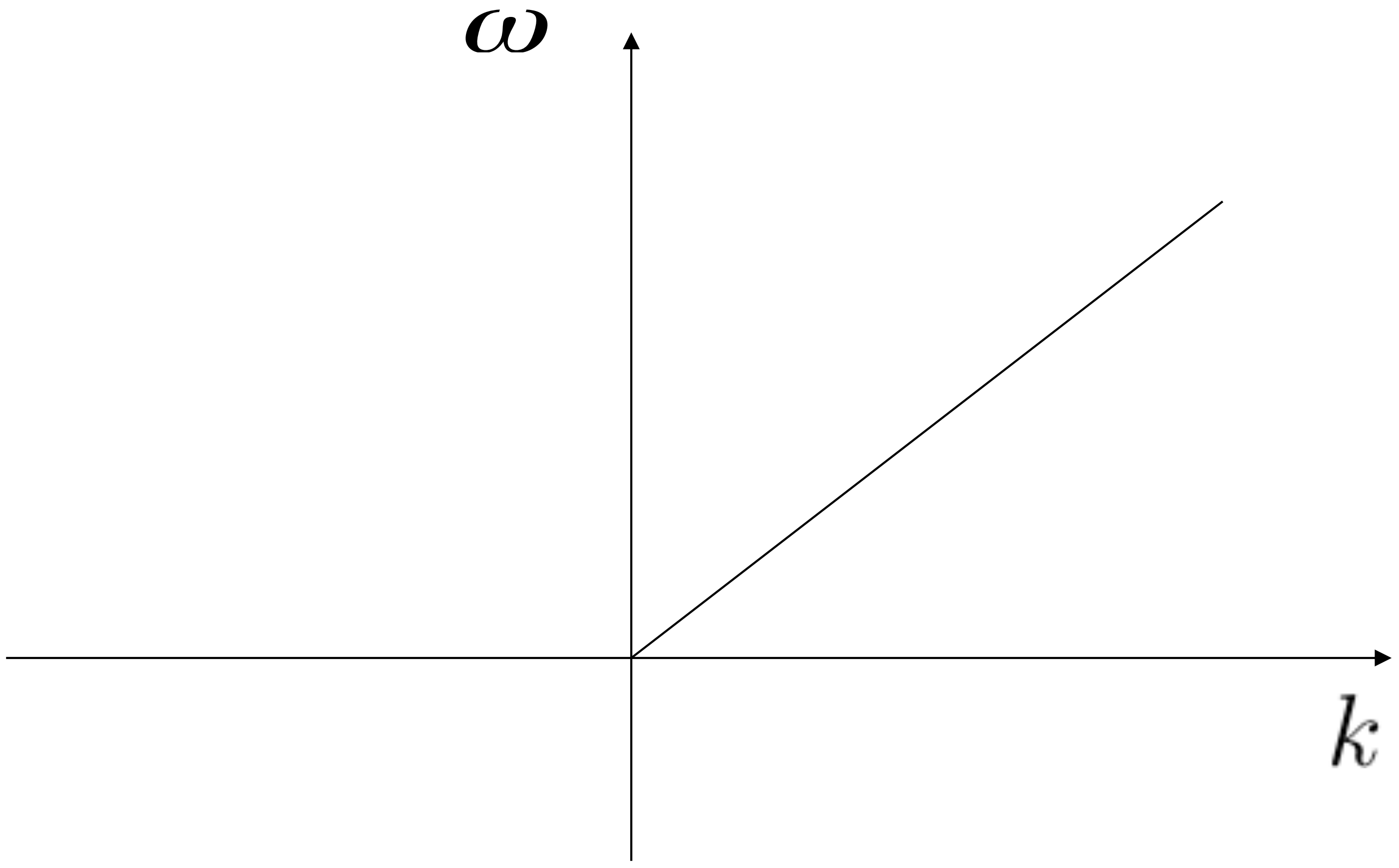
First baroclinic mode:

$$c_1 = 2 \sim 3m/s$$

Phase speed and group velocity of Kelvin waves:

$$c_p = c_g = c$$

Non-dispersive. Both phase and energy propagate eastward. Exist for all frequencies



b) Solution:

$$\eta = e^{-\frac{\beta y^2}{2c}} G'(x-ct)$$

$$u = \frac{g}{c} e^{-\frac{\beta y^2}{2c}} G'(x-ct)$$

$$v = 0$$

y>0 N.H.

EQ y=0

y<0 S.H.

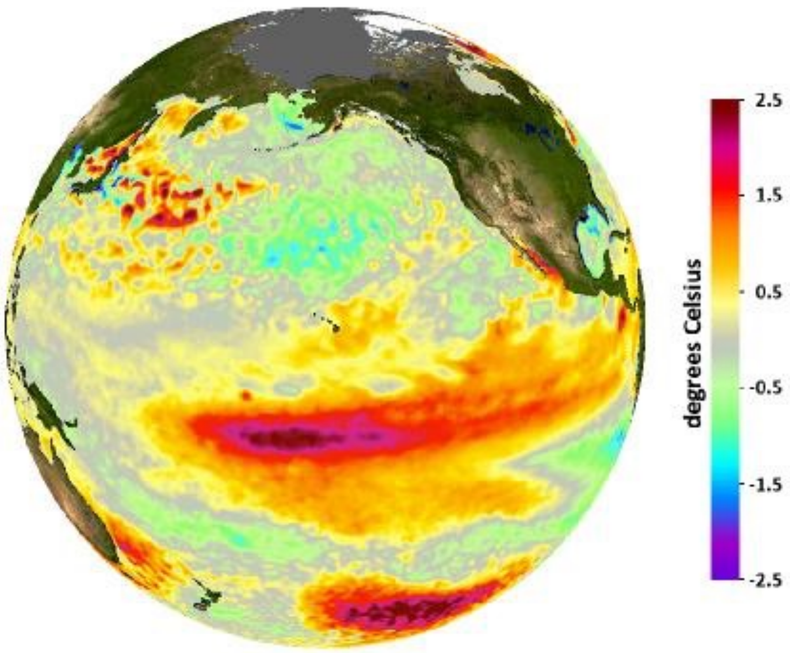
E-folding scale: $a = \sqrt{\frac{2c}{\beta}}$

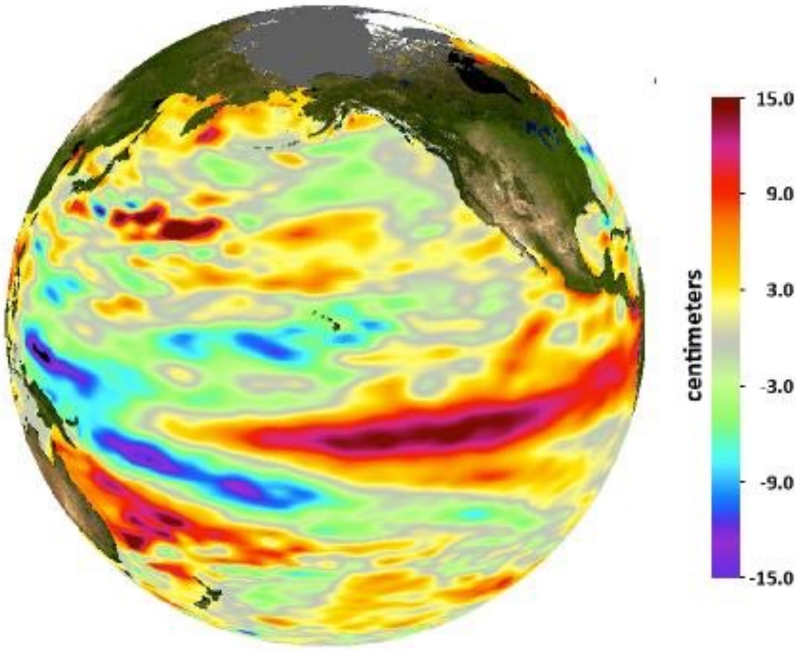
Equatorially-trapped: due to β

c) **Symmetric about the equator (u and p/sea level)**

d) ***Forcing. Changing winds with time – symmetric about the EQ.***

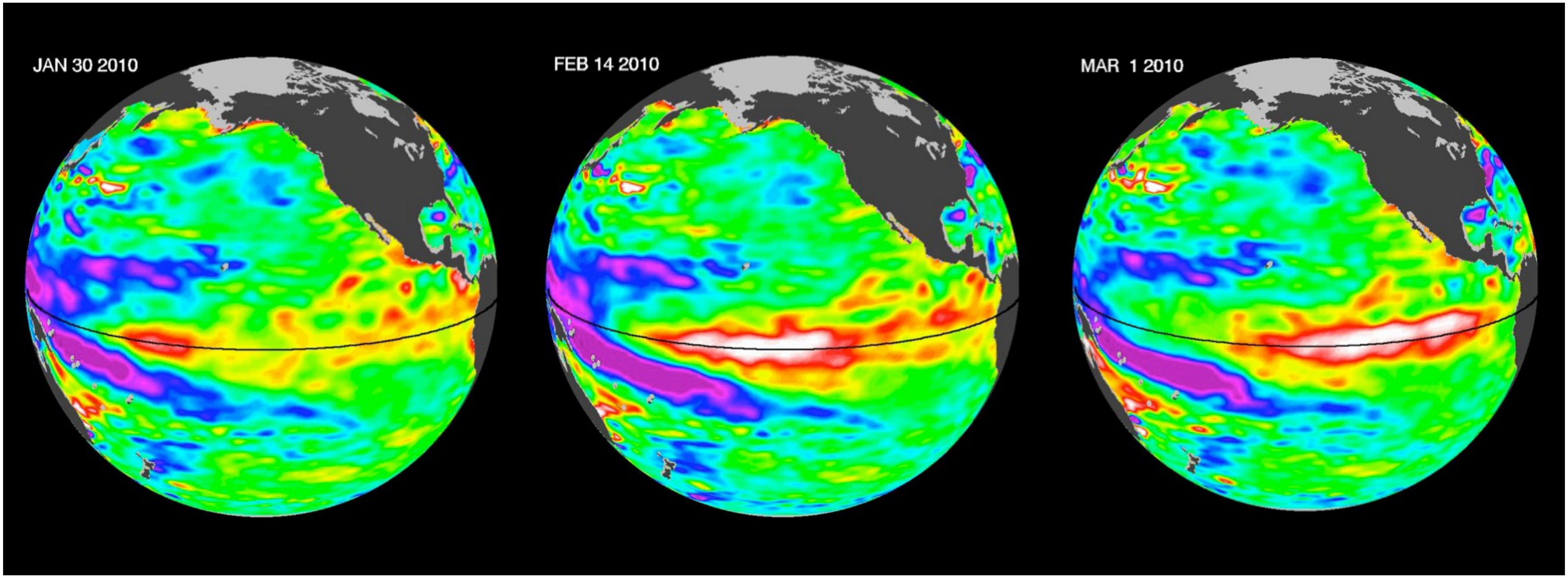
Equatorial Kelvin wave structure


Sea Surface Temperature Deviation From Normal


Sea Surface Height Deviation From Normal

Red contours: ***Symmetric about the equator for Sea surface height (SSH)***

Satellite-observed sea surface height anomalies

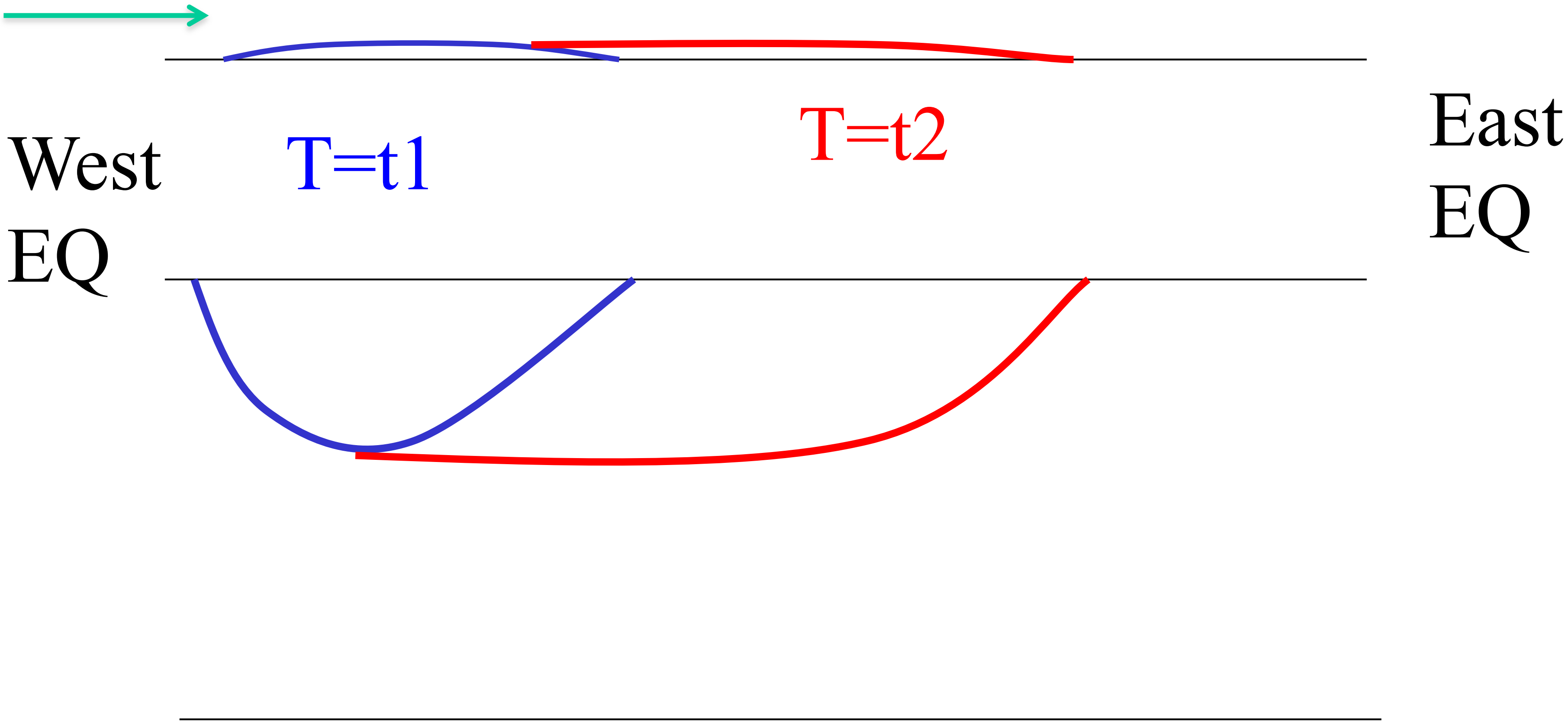


Propagation: direction?

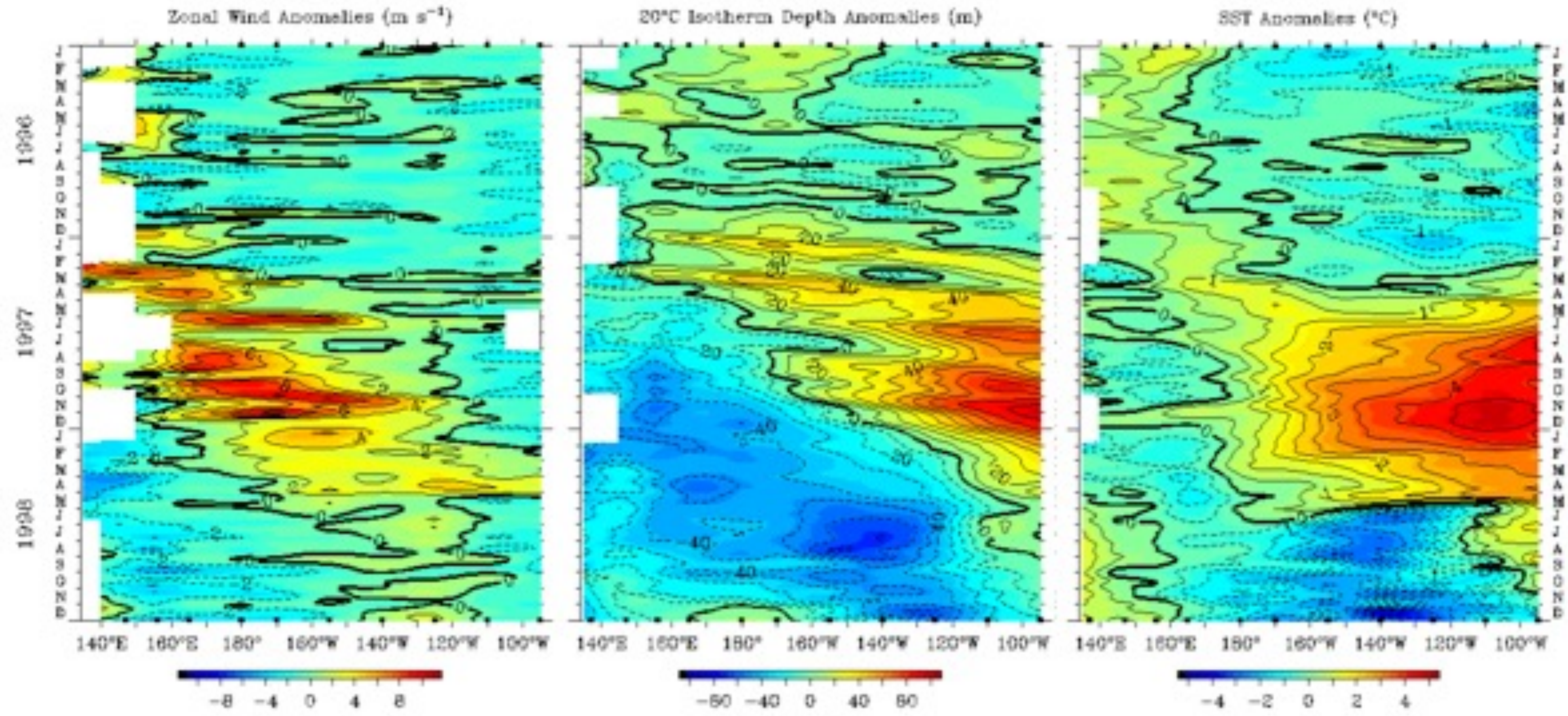
Eastward

Westerly wind burst (WWB) associated with the Madden-Julian Oscillation (MJO):

Equatorial (EQ) westerly wind burst



WWB, EQ Kelvin wave & onset of the 1997 El Nino



McPhaden 1999, science.